

The Function of the Second Postulate in Special Relativity

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Abstract

Many authors noted that the principle of relativity together with space-time homogeneity and isotropy restrict the form of the coordinate transformations from one inertial frame to another to being Lorentz-like. A free parameter in these equations, k , plays the part of c^{-2} in special relativity. It is usual to claim that k is determined by experiment and hence, that special relativity does not need the postulate of constancy of the speed of light. I analyze how one would go about determining k empirically and find that all methods suffer from severe problems without further assumptions, none as simple and elegant as the postulate of constancy of the speed of light. I conclude that while the formal structure of the transformation equations can be determined without appeal to the second postulate, the theory is left without physical content if we ignore this postulate. Specifically, evaluating k requires creating a signal that travels identically in opposite directions or ensuring that such a signal exists. Assuming this property about light is sufficient to obtain special relativity, although the condition is logically weaker than Einstein's postulate of the constancy of the speed of light. The physical core of the second postulate, therefore, lies in its assurance of the isotropy of some signal.

Keywords: Special Relativity, Lorentz Transformations, constancy of the speed of light, Second Postulate, Electromagnetism

1. Introduction

In his famous 1905 paper, Albert Einstein based the theory of special relativity on two principles, or postulates (Einstein, 1905):

1. [Principle of relativity] The laws governing the changes of the state of any physical system do not depend on which one of two coordinate systems in uniform translational motion relative to each other these changes are referred to.

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2. [Principle of the constancy of the speed of light] Each ray of light moves in the coordinate system “at rest” with the definite velocity V independent of whether this ray of light is emitted by a body at rest or in motion. (Einstein, 1905, p. 895, English translation from The Collected Papers, Vol. 2, 1989, p.143)

Homogeneity of space and time and isotropy of space were implicitly assumed, and were later explicitly added to the list of postulates.

It was noted very early on that the second postulate seems to hold a very different status from the other assumptions. While the principle of relativity, spacetime homogeneity and spatial isotropy are all general symmetries of nature, the second postulate appears to be a very particular and contingent claim about a specific phenomenon.

Starting with Ignatowski (Ignatowski, 1910, 1911), and Frank and Rothe (Frank and Rothe, 1911), many researchers questioned the necessity of the second postulate (e.g., Weinstock, 1965; Mitvalsky, 1966; Lévy-Leblond, 1976; Srivastava, 1981; Mermin, 1984; Schwartz, 1984; Sen, 1994; Pal, 2003, among others). Their aim was expressed very explicitly by Lévy-Leblond (1976):

...I intend to criticize the overemphasized role of the speed of light in the foundations of the theory of special relativity, and to propose an approach to these foundations that dispenses with the hypothesis of the invariance of c . (Lévy-Leblond, 1976, p. 271)

The physical motivation behind these derivations was expressed most clearly by David Mermin (1984):

Relativity is not a branch of electromagnetism and the subject can be developed without any reference whatever to light. (Mermin, 1984, p. 119)

A.M. Srivastava (1981) sums up the situation before beginning his own derivation of the Lorentz transformations by stating that:

It has been pointed out by many authors that the postulate of the constancy of the speed of light is not necessary for arriving at the space-time transformations in special relativity (Srivastava, 1981, p. 504)

What these various authors actually derive are “generalized Lorentz transformations”, i.e., the coordinate transformations obtained from the spacetime symmetries and the principle of relativity alone. These are (see the derivation in e.g., Drory, 2014):

$$\begin{aligned} x' &= \frac{(x - vt)}{\sqrt{1 - kv^2}} \\ t' &= \frac{(t - kvx)}{\sqrt{1 - kv^2}} \end{aligned} \tag{1}$$

Here, x', t' refer to the coordinates of some event observed by an observer S' . This observer moves at a constant velocity v with respect to a second observer S , who ascribes to the same event the coordinates x, t .

From these we obtain immediately the rule for the addition of velocities:

$$u' = \frac{v + u}{1 + kuv} \quad (2)$$

which describes the velocity u' measured by S' for a body that moves at a velocity u in the x -direction, with respect to S .

The generalized transformations contain an undetermined universal parameter, here denoted k , which is equal, in standard SR, to $\frac{1}{c^2}$. A.M. Srivastava's attitude seems quite common, when he notes:

As we know, the experiments show that $[k^{-1/2}]$ has a finite value which is equal to the value of the speed of light. (Srivastava, 1981, p. 505)

Similarly, Mermin (1984) explains:

From this point of view, experiments establishing the constancy of the speed of light are only significant because they determine the numerical value of the parameter k . (Mermin, 1984, p. 119)

In the context of these derivations, it thus appears that the content of the second postulate is merely a report of the experimental value of $k^{-1/2}$, which just happens, utterly contingently it seems, to be the velocity of light. Neither light itself, however, nor any other signal propagating at $k^{-1/2}$ plays any role in these derivations, it is claimed. Mermin goes even further in declaring:

It is not, however, necessary for there to be phenomena propagating at the invariant speed to reveal the value of k . (Mermin, 1984, p. 123)

Now these transformations are undoubtedly of great interest. It is hardly obvious that the relativity principle and the space-time symmetries by themselves restrict the possible coordinate transformations to just two types: Galilean and Lorentz-like. Indeed, this is the point justly stressed by Pal in his own derivation (Pal, 2003). Whether these derivations succeed in their avowed aim is less clear, however. An early opposing view was expressed by Pauli. After reviewing the derivation of these transformations, he wrote:

Nothing can naturally be said about the sign, magnitude and physical meaning of $[k]$. From the group-theoretical assumption, it is only possible to derive the general form of the transformation formulae, but not their physical content. (Pauli, 1958, p.11)

Unfortunately Pauli did not elaborate on this claim, which may have let subsequent authors to either believe that they had managed to overcome his criticism or else to dismiss it as unfounded. I share Pauli's opinion, however and the present work can therefore be seen as an elaboration of why the physical content of the generalized transformations requires something like the second postulate.

In a previous work, (Drory, 2014), I have argued that as a physical theory, special relativity must be explicitly distinguished from Newtonian mechanics, since these two theories imply different ontologies, different procedures for reproducing experiments, etc... This means that the cases $k = 0$ and $k \neq 0$ represent two different theories, each distinguished from the other by an additional postulate regarding the value of k . While that value must of course be determined empirically, the distinction between the vanishing and non-vanishing cases is so fundamental that it must be included among the postulates of the theory before we can speak of the resulting structure as a well defined physical theory at all.

The present work attacks the problem from a different point of view. Even if we suppose that $k \neq 0$, and consider this, for some reason, a more acceptable postulate than the constancy of the speed of light, the transformations are still not well defined, for we must be able to determine in principle the value of k . This is taken to be a trivial operation by most authors on the generalized Lorentz transformations (N David Mermin being a notable exception and I shall analyze his insights in detail), but I shall argue that if $k \neq 0$, its value cannot be experimentally determined without additional information and that the type of information required is precisely of the kind offered by the second postulate.

It is important to note that $k \neq 0$ and $k = 0$ suppose different preconditions of experiments. As we shall see, many experimental procedures call for a set of synchronized clocks. If $k = 0$, there is no problem setting up such a set. The rate of flow of time is absolute and clocks may thus be transported at arbitrary speeds after they are synchronized at a common location. Not so if we assume that $k \neq 0$. The assumption itself implies that clocks in motion tick at a rate that depends on their velocity so that synchronization by transport is no longer an available option. Thus, if a certain measuring procedure of k requires, e.g., a set of synchronized clocks, we need to assume a priori whether k vanishes or not. Since we do not know yet the value of k , the default assumption should be that it is non-vanishing.

This assumption differs completely from the need to elevate the non-vanishing of k to the level of a postulate if we seek to determine our theory's consequences. The decision to treat k as non-zero as a precondition of experiments must be considered tentative and does not preclude the possibility that we will discover that it vanishes after all (to the extent that experiments can determine this). The cases $k = 0$ and $k \neq 0$ are not on the same level when it comes to preconditions of experiments. To take once again the case of clock synchronization as an example, any procedure consistent with the assumption that $k \neq 0$ will also be consistent with the possibility that $k = 0$, but the reverse is not true. Synchronization by finite velocity transport is only acceptable if k vanishes, but the Einstein synchronization procedure, for example, is valid even if $k = 0$.

Thus, as far as experimental preconditions are concerned, the assumption $k \neq 0$ is more general than $k = 0$, and therefore, as noted above, does not preclude that we should discover that k vanishes after all. The assumption that $k \neq 0$ should be viewed as no more than possibly excessive caution against experimental biases. Hence, although we must set up the experiments *as if* $k \neq 0$, in fact we make no assumptions as regards the actual value of k . It may be zero, or high or low or any value at all.

With this in mind, I shall argue that measuring k experimentally runs into serious difficulties if we do not use something akin to the second postulate. The structure of the paper is as follows: After setting up some ground rules in section 2, section 3 analyzes the possibilities of measuring k from the time and space transformations. I shall argue that these require a network of synchronized clocks. After examining several synchronization methods, I conclude, in section 4, that all require a method of signaling at equal speeds in opposite directions. Section 5 examines the relevance of the notion of isotropy to this enterprise, and section 6 argues that the one-postulate program cannot guarantee a method of synchronizing clocks. Mermin's procedure for measuring k , which bypasses the need for synchronized clocks, is analyzed in section 7 and found to suffer from identical problems. Section 8 presents a last ditch attempt, using the Doppler effect, but this also fails. Section 9 then considers the question of identifying inertial frames from the point of view of the current discussion. Section 10 then presents some general considerations why special relativistic theories need a postulate about the properties of specific signals. I show there that one can recover the standard theory of special relativity by adding to the fundamental symmetries a postulate on the isotropy of propagation of light. Although logically weaker than Einstein's second postulate, it is sufficient to derive the invariance of the speed of light. The paper then closes with a brief summary of the main points.

2. Ground rules

A proper evaluation of the claims made by supporters of the one-postulate stand requires setting up some basic rules of analysis.

First, these claims concern the logical structure of the theory, which is assessable in retrospect. Thus we ignore the historical role that the second postulate played in Einstein's thought.

Instead, the project of single-postulate supporters can be described thus: Assume only the following three postulates:

A. Homogeneity of space and time: The laws of physics are invariant under a shift in the origin of space or time coordinates.

B. Isotropy of space: The laws of physics are invariant under a rotation of the axes used to described them.

C. The principle of relativity: The laws of physics are invariant when referred to different systems of reference, which are in uniform motion with respect to one another.

From these assumptions alone, derive the special relativistic kinematics up to a numerical constant that is determinable in principle *without invoking further physical knowledge*.

The italicized part above is essential to the evaluation of the enterprise. By claiming that special relativistic kinematics can be derived from just the assumptions mentioned above, we bar ourselves from using additional theoretical or empirical data. Were it not the case, one could conceivably claim that the principle of relativity itself need not be counted among the theory's assumptions, since it can be obtained by appeal to established tradition, mechanics and /or countless experiments, going back to the time of Galileo and Descartes.

Of course, historically, that principle needed reiterating in its general form because Maxwell's electrodynamics seemed to conflict with it. But under the rules of the game we are trying to play, electrodynamics must also be ignored. Were this not the case, the second postulate could be said to follow from Maxwell's theory just as well, which is of course the way Einstein approached his derivation.

Thus, what is at stake here is whether we can derive in a meaningful way special relativistic kinematics from just the three assumptions A-C. "Meaningful" implies, among other things, that we can make sense of all the quantities that appear in the equations. In particular, we must be able to devise a method by which we measure k to arbitrary precision.

To this end, our main tool is the gedanken experiment, a set-up that serves a logical purpose but not necessarily a practical one. A quantity is said to be empirically determinable in the gedanken sense if we can imagine some procedure that would yield its value, provided we ignore technical difficulties and experimental errors, but as long as we do not break any physical laws or ignore essential effects. It is sometimes a very thin line deciding whether an obstacle is a technical limitation or a fundamental one, nowhere more in evidence than in the long history of seeking *perpetuum mobile* machines, which all ignore fundamental effects by reclassifying them as technical difficulties. Thus extreme care is required in devising gedanken experiments and in analysing their results.

Gedanken experiments are supposed to take place with infinite precision, as far as technical limitations apply. In the present context, we wish to obtain an ideally precise value for k .

Finally, a word about what it means to derive special relativity. All derivations, whether based on the second law or not, have this in common that they concern the logical structure of the theory, not its correctness. It may be that the principle of relativity does not in fact hold. It may be that the velocity of light is not invariant, and that some other velocity is. It may be that there is no invariant velocity. At no stage does any derivation consider whether the postulates are empirically adequate. We know in fact that the speed of light is only invariant in matterless space, where gravitational effects can be ignored. But this invalidates all the derivations, or none. It is therefore irrelevant to the present discussion. Similarly, any discussion of the limitations of special relativity induced by quantum effects is out of place, unless it can be shown to directly impact one or more of the postulates of special relativity. Thus, for ex-

ample, the notion that objects have precisely definable trajectories irrespective of their scale may be incorrect, but this has no relevance here. For the same reasons, the criticism I will level at the derivations that attempt to bypass the second postulate ignore the domain of adequacy of postulates and theory both. In other words, I will consider the postulates to be true, even if we happen to know that in fact they are not or that they have a limited domain of validity.

3. Finding k from the transformations

One possibility of measuring the value of k is to use the generalized Lorentz transformations themselves. The transformations offer two direct methods of obtaining this value, one from the contraction of lengths, the other from time dilation.

Let us start with the contraction of lengths. As in SR, we assume a rod with rest length L_0 placed in a S' -reference frame moving at velocity V with respect to an observer S . To measure the length of the rod, S simultaneously measures the positions of the rod's end points. The length measured by the observer S is then

$$L_0 = \frac{L - vt}{\sqrt{1 - kV^2}} - \frac{0 - vt}{\sqrt{1 - kV^2}} \Rightarrow L = \sqrt{1 - kV^2} L_0 \quad (3)$$

There are several problems arising with this proposal, the more serious ones having to do with the general question of setting up a proper system of units for measuring length. I shall deal with these in an upcoming work (Drory, 2014b). For now, suffice it to note that this determination of k requires measuring the length of a moving object, which, as stressed by Einstein, requires that we locate the positions of the object's end points simultaneously. This means that we need a set of synchronized clocks, so the question becomes how to generate such a set.

The exact same problem arises if we try to use time dilation. The rates of two clocks, each ticking in its own rest frame, are related, according to the generalized Lorentz transformation, by the formula

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - kV^2}} \quad (4)$$

But the problem now is to determine how the two readings are to be compared.

The first thought might be to use a form of the twin paradox. Let us synchronize the clocks of S and S' at a given moment $t = t' = 0$. Let S' run in a very large circle at a constant speed V . When the clock in S' eventually returns to face the clock in S , the two elapsed times can be compared. This procedure encounters the same problem as in the original twin paradox, however. Because S' is rotating, it is no longer inertial, and the symmetry between the systems is broken. The relativity principle can no longer ensure that both clocks run identically in their own rest frames, therefore.

Of course, accelerated systems can be treated in the framework of relativity-like theories, by the use of instantaneously comoving inertial reference frames. But the problem goes beyond purely kinematical effects. Clocks have internal mechanisms that are essential to their validity as time keepers. Accelerations entail fictive forces and these might influence the clock's mechanism. The possible effects of acceleration on any given clock must be determined according to the specifics of the mechanism.

These concerns are not hypothetical. The 1971 Hafele-Keating experiment in which cesium clocks were flown around the world and compared with a clock that had remained on the ground is a good showcase for the problems involved in such a method (Hafele and Keating, 1972). Because the planes were flown at high altitude, Hafele and Keating had to take into account the gravitational influence on the running of the clocks. It turns out that this effect is of a similar magnitude as the kinematic effect due to the plane's velocity. Furthermore, the clock on the ground is not in an inertial system either, because of the daily rotation of the earth, so all clocks must really be compared to an imaginary clock at the center of the earth. The difference in the ticking rate of the clock on the plane and on the ground then depends on the direction of flight of the plane, further complicating the comparison.

The point here is not to analyze the difficulties of the Hafele-Keating experiment, but rather to note how complicated it is to take into account the effects of the acceleration, which by the principle of equivalence are of a nature similar to those of gravitational fields. In the case of the Hafele-Keating experiment, the effects had to be calculated out before a proper comparison of the clocks could be performed in order to check just the special relativistic time dilation. Yet clearly, such calculations cannot be performed without a previous knowledge of the value of k . For example, we have no way of knowing how important the effects of the acceleration are compared to the kinematical effects without knowing k at least to a degree of magnitude. Worse, whatever calculations we perform can never count as a direct determination of k because we will have to depend on many more assumptions (in the case of the Hafele-Keating experiment, for example, general relativity was required in order to take into account the gravitational effects on the two clocks).

Consider for example Cesium atomic clocks. Their operation depends on the behavior of the hyperfine structure of the atoms, on the interactions of the atoms with magnetic fields (used in the detectors) and on microwave generators used to provide the exciting frequencies. To compare correctly the difference in rates of the clocks S and S' , we need to know how each of these components are affected by accelerations (fictive forces). This requires considerably more than just the postulates A-C above, even in Gedanken settings.

In comparison, the second postulate solves the equivalent problem in standard special relativity by offering a simple (gedanken) time-keeping device, namely Einstein's light-clock. This device's proper function as a time-keeping device is ensured by the second postulate itself. But if we reject the postulate, we must replace it by some other assumptions regarding possible clock mechanisms, so that we can compare the workings of two clocks in asymmetrical

frames.

The simplest solution to this problem is to sidestep it entirely by keeping both S and S' inertial. The principle of relativity itself then ensures that the two clocks function identically without any need to enter into specific details. This means that the two frames must move in straight lines, which implies that their origins can only coincide once. As before, we use this moment to synchronize both clocks so that $t = t' = 0$ at this time. In order to compare the ticking rates, let us assume that the origin of S' reaches the location x in the S-frame at a time t in that frame. At this moment, the clock in S notes the time shown by the clock in S' (it may take a picture of it, for example), while also noting the moment t in its own frame. The time t' displayed by the clock in the S' frame is then

$$t' = \frac{t - kVx}{\sqrt{1 - kV^2}} \quad (5)$$

Now to extract k from this we need to know the velocity of the S' frame, V . But this is easily obtainable since clearly $V = \frac{x}{t}$, x and t being the position of the clock in the S frame and the time it recorded when noting the value of t' . Substituting this into Eq.(5) then yields

$$t' = \sqrt{t^2 - kx^2} \quad (6)$$

Since we do not need to accelerate any object, this method of measuring k at least bypasses all the dynamical concerns arising from that process. The determination of x represents a rest-length measurement in S, which only requires a ruler, so there are no concerns there, but the measurement of the time t still requires two synchronized clocks (one at x , the other at the origin). Just like the measurement of k from length contraction, we are back to the need for a synchronization procedure. The Einstein synchronization is based on the second postulate. What are the alternatives?

4. Synchronizing Clocks

As mentioned in section 1, the finiteness of k has implications about our choice of clock-synchronization method. Synchronizing clocks when they are together and then transport one of them to another position is an unacceptable procedure if we wish to make no assumptions regarding the value of k , other than its possible non-vanishing.

The literature on the foundations of special relativity often mentions slow-transport synchronization, i.e., the idea of moving the second clock to a new position at an arbitrarily small speed. This notion has always seemed very odd to me, in the context of foundational discussions. When discussing such questions, we routinely enter the Gedanken mode, in which we assume infinite experimental precision and hence also require infinite precision in our results. Slow-transport is singularly ill-adapted to the Gedanken spirit of measuring fundamental quantities *in principle*, because its whole essence relates to more or

less acceptable approximations. While formally we can imagine a series of procedures in which the transport speed is continually reduced, it is quite unclear how this is supposed to translate into an experimental procedure. Recall that Gedanken experiments must be implementable in principle, i.e., without violation of physical laws or essential aspects of the procedure. Yet *no matter how slow the transport*, the resulting clocks will always be out of synchronization to some extent. This is a violation of the whole spirit of Gedanken procedures. Of course, slow transport may be an excellent choice *in practice*, if we keep track of the degree of asynchronicity introduced and take it into consideration when estimating the experimental error. But thought experiments are supposed to contain no experimental error at all, so they marry badly with the very idea of slow-transport synchronization.

In the present case, the situation is made considerably worse by our ignorance of the value of k , which is precisely what we are trying to measure. Now how slow is slow enough for any given precision cannot be determined in advance if we do not know the value of k and hence have no way of estimating the coefficient $\sqrt{1 - kV^2}$. Once again, this seems like a singularly bad approach for answering questions of principle. In the end, the only way to make use of slow-transport synchronization is by trial and error. One must venture some initial guess as to the value of k , choose a slow “enough” velocity, according to some predetermined required precision, transport the clocks some distance, then back again at an identical speed to compare them and see if the synchronization holds to the required precision. There is no way to know in advance if the result is satisfactory since the initial guess as to the value of k is completely arbitrary. If the synchronization is insufficient, we must reduce the transport speed and repeat the procedure, possibly over and over again. While there is nothing wrong with this method, and in practice something of the sort is perfectly acceptable for various purposes, it is singularly inadequate to answer questions of principle as it never offers a way of establishing synchronicity per se. Instead it only offers synchronization up to some pre-established precision of measurement, which is not what we set out to obtain in the first place, though, again, it might suffice to measure the value of k to some approximation, albeit in a remarkably inelegant way due to the necessity of repeated trial and error.

Yet even if slow transport is used, and we are willing to give up on elegance or infinite precision in our procedural definitions, we have still not cleared all the hurdles. To assess whether the transport speed is low enough, we must bring the two clocks together or else find a way to establish synchronicity from afar. But if we can perform the latter operation, we have no need of slow transport in the first place, as we can just set the clocks manually until they are synchronized. Thus, slow transport requires that the two clocks meet again. One option is to move one clock in a circle, but that brings us back to the problems of acceleration discussed in the previous section. The second option, as described above, is to move the clock aside and then back at an identical speed in the opposite direction.

And this is precisely where most methods of measuring k hit a snag. For if we could do that, i.e., move a clock in two opposite directions at the same speed,

we would again have no need of slow-transport at all. Instead we could use a simpler procedure suggested by N. David Mermin (Mermin, 1984, p. 124, note 5).

Suppose that we want to synchronize two clocks, one located at the origin and the other at a position $x_2 = L_0$. Place both clocks at the midway position, $x_1 = L_0/2$, and synchronize them. Assume the two clocks are mounted on identical rails with identically built motors that can move them. From the midway position, transport one clock to the origin and the other to the position $x_2 = L_0$. We do not need to know how the clocks move exactly, nor what are their velocities, or indeed whether they are moved uniformly or not. All we need know is that they move identically, which ensures that they remain synchronized.

And now we encounter the problem that we shall meet again and again. How can we be assured that the two clocks move identically? Note that we do not care how they move, but we do require that their motion should be the same. At first sight, this might seem easy. Build two identical motors and rails. To make sure that they are identical, run two clocks along the two rails when the motors work in the same direction instead of in opposite directions. If the moving systems are identical, the two clocks will be in identical positions at every instant. We do not need to know what that time is, so we do not need clocks for this verification. It is enough that whenever one clock passes a given point in space, the second passes that point at the same time. Once this is determined, rotate the rails and motors of one clock by 180° and use that to move the second clock to its required position after it has been synchronized with the first.

But how do we know that the rotation hasn't influenced the rate at which the motors transport the clock? A clock, for example, is also a mechanical device of some sort and we know that rotating a clock will destroy its synchronization with an unrotated clock. Therefore, rotating a mechanical device, in this case a clock, can influence its working. How can we be sure that something of the sort does not happen to the motor? Barring a complete description of every detail and structure of the transportation system, and an equally complete understanding of every physical effect involved, we cannot be certain of this.

To ensure that the two clocks move identically, we must find a way to build two identical mechanisms that work in opposite directions without physically rotating one of them. Here, spatial isotropy seems to come to the rescue. Instead of rotating the mechanism itself, rotate its theoretical structure in space. In other words, rotate the plans of the mechanism and build the moving mechanism according to the rotated plans. Nothing would be physically moved, and spatial isotropy would ensure that the rotated structure functions identically to the original one.

But what do we mean when we say that spatial isotropy ensures this result?

5. Two meanings of isotropy

Isotropy is one of the fundamental symmetries used to derive the Lorentz transformations, in either their standard or their generalized form. It would seem thus that we are on solid grounds in invoking it for the Mermin synchronization procedure.

Unfortunately, the term “isotropy”, and more generally the notion of (space-time) symmetries, cover two related but nevertheless distinct concepts. The first is a general principle, according to which the form of the laws of physics possess certain invariance properties. In the case of isotropy, this means that the laws exhibit no preferred directionality (rotational invariance). This can be called “nomological symmetry” or in this case, “nomological isotropy”. The second possible meaning of “isotropy” is that we see no preferred directionality in a specific experiment performed under well-defined conditions. In this case isotropy refers to invariance under an actual physical rotation of the system. Such a property may be termed “systemic isotropy” or more generally “systemic symmetry”.

5.1. *Nomological symmetries*

Nomological symmetry is an abstract, mathematical property of the laws. It states that the expression of a law, or a number of laws, is covariant with respect to some transformation, e.g., of the coordinate system. Thus, when we claim that the fundamental laws of mechanics, for example, exhibit rotational invariance, we mean that these laws retain their mathematical form under a rotation of the axes. When all vector components are appropriately recalculated in the rotated frame, their mathematical relations look the same as they did before the rotation.

The rotation under question here is a rotation of the coordinate system, not a physical rotation of the system. Such a mathematical transformation is of course equivalent to a physical rotation of the system in the opposite direction while keeping the axes unchanged, but it is crucial to understand what we mean here by “system”. A rotation of the axes is a global transformation, not a local one. It is thus only equivalent to a global rotation of the system. In principle, therefore, a rotation of the axes is equivalent to a physical rotation of the entire universe in the direction opposite the rotation of the axes. Naturally, we expect that rotating the entire universe is geometrically meaningless (but not necessarily dynamically so, as suggested by Mach’s principle). This merely reflects our view that the orientation of coordinate systems is arbitrary and should carry no meaningful content. That is the content of nomological isotropy.

This seems at first sight to be too confining a definition. Noether’s theorem, for example, applies to any Lagrangian that is invariant under a continuous symmetry. Nowhere does it require that this Lagrangian should describe the whole universe. But this is only because here as with all their uses, nomological symmetries apply to abstract systems, not to actual objects. In Noether’s theorem, a rotation still means a mathematical transformation of the coordinates, not a physical rotation of the system or of part of it. By limiting ourselves to a

specific Lagrangian, we are in effect creating a toy universe which contains only the terms referred to in this Lagrangian. We are thus imagining that the rest of the universe has been erased, and that the whole of actuality is described by the terms we include in the Lagrangian. Should we desire to consider a physical rotation of the system, we must apply it to the entirety of our toy universe. Thus we are still rotating the “entire universe” but this is a fictional universe that contains only a limited number of objects.

The question whether such a fiction is useful or not is a different matter. Clearly, when we discuss nomological isotropy in this sense, we are imagining that every *relevant* factor has been rotated, or in other words, we are imagining that the Lagrangian contains every factor of import to the phenomenon we are investigating.

Consider for example, the description of ballistic motion and let us ask whether this problem exhibits invariance under horizontal rotations (clearly, the direction of the force of gravity selects the vertical component as special and full three-dimensional isotropy does not hold). The general laws of dynamics are isotropic, so we might expect that the answer to this is positive, but the question centers on what exactly we mean by horizontal isotropy, which depends on the level of description that we wish to include, i.e., how rich we wish to make our toy universe. This in turn depends on the level of precision we require of our description.

The simplest level will only consider the force of gravity (whether we take into account its height dependence or not). In this approximation, the laws are horizontally isotropic, so that objects thrown from identical height at identical velocities will land identical distances away from their initial point. I assume here that the azimuth angle of throw is identical, but that the horizontal direction of the initial velocity may differ. We know, however, that this description is only valid for short distances and short times of flight. In actuality, objects thrown northwards and eastwards will exhibit a different behavior because of the Coriolis force generated by the earth’s daily rotation. To take this into account in the laws requires the introduction of the earth’s angular velocity vector, which identifies the direction of the Earth’s spinning axis as a special direction. In this case, the existence of nomological isotropy is conditional on the understanding that when we rotate the axes, the angular velocity vector is also rotated in the required way. This means that nomological isotropy is here equivalent to the physical rotation of a system that includes our planet. In effect, the direction of projection with respect to the geographic cardinal points remains invariant. We may rotate the direction in which we throw the object by 90° , but if we rotate the earth by the same amount, east will not turn into north. It will remain east, albeit a rotated east with respect to a coordinate system exterior to earth.

For higher precision, however, this might not suffice. The equations that include the influence of earth’s self rotation are good only to the degree that the rotation of the earth around the sun is negligible. Thus, for most earth-bound projectiles, this is likely to suffice, but objects thrown from earth into space might feel a non-negligible effect from the Coriolis force (for example) due to

our planet's yearly rotation. In this case, we need to include the sun into the relevant system. Nomological isotropy is then valid only if under a rotation of the axes we also rotate the angular velocity vector of the earth around the sun. When considering now what physical rotation might be equivalent to this mathematical transformation, the answer is that we must consider a rotation of the entire solar system. As more and more precision is required, we might need to include our arm of the galaxy, the galaxy itself, the local group and so on. While we need not consider these levels in practice, if we ask a question of principle, such as in a gedanken-situation in which infinite precision is sought, then the isotropy of the laws can only be equivalent to a physical rotation of the entire universe.

5.2. *Systemic Isotropy*

The preceding argument makes it clear that nomological isotropy is only meaningful in terms of abstract properties of mathematical descriptions of systems. When we consider the actual meaning of isotropy in empirical situations, we clearly ask a different question. Suppose that we are performing some measurement, with an apparatus that contains an elongated arm of some sort. We now wonder whether the result exhibits some directionality. In order to check this, we physically rotate our apparatus, which we suppose to include the object on which the measurement is performed, without changing any other parameters. The question now is whether we shall obtain the same result as before. If we do, then our system exhibits a kind of rotational invariance that we may call "systemic isotropy", or in general "systemic symmetry".

The essential difference between this and nomological isotropy is that here, the operation under which the system is invariant is a physical rotation of the local system investigated, not an abstract rotation of the axes. Because physical rotations cannot be performed on the entire universe, they necessarily apply to a small, local set of objects, on which we have physical control. The rotational invariance of these systems depends on the degree to which they are dependent on the external world or influenced by it.

Let us return to the simple example of ballistic motion. We cannot as a matter of fact rotate the earth or the solar system. Thus, physical rotations are limited to the direction in which the object is thrown and possibly some local conditions (e.g., the experiment is performed in a sealed vacuum cube that may also be rotated). If our experiment is sensitive enough to pick up the effects of the Coriolis force, we shall not have systemic isotropy. In applying nomological isotropy, we rotate all relevant vectors, including the angular velocity of earth, because we are actually rotating axes, not systems. The laws are covariant under such a transformation. But physically, we can only rotate some of the vectors, e.g., the initial velocity of the object. The planet's angular velocity is unavailable to manipulations.

Systemic isotropy concerns a physical transformation performed on the system, or a physical symmetry that the system possesses, not a mathematical manipulation of the axes used to describe this system. Thus, systemic invariance is entirely an empirical matter to be decided by comparing the results of

experiments that differ only in the physical orientation of the apparatuses used to perform them.

Systemic and nomological symmetries are completely separate and independent properties, neither of which entails the other. Systemic symmetry is a contingent property and we have many cases of events that exhibit no systemic symmetry, though the general laws of physics that determine them are nomologically invariant. Conversely, it is certainly possible that although a certain class of phenomena is devoid of nomological symmetry, a specific system, through some accidental cancellation of effects, for example, should exhibit this symmetry systemically. For example, there is no law to the effect that every object should have an existing mirror image. The number of right-handed corkscrews is vastly superior to the number of left-handed ones. But a system of socks does usually exhibit such a symmetry so that at least at the manufacturing level, every sock of a given pattern usually possesses a mirror-symmetry twin (washing machine jokes aside).

The structure of special relativity itself reveals this twin aspect, for the postulate of isotropy is clearly meant to be nomological. In the derivation of the Lorentz transformations (whether generalized or not) it is applied to the axes themselves, or equivalently, to the frames of reference, where these are supposed to extend over the entire universe. Again, whether such a supposition holds in practice is a different matter. General relativity restricts the postulates of special relativity to local frames only. This represents a limitation on the empirical adequacy of special relativity in our world, not a limitation on the use of these postulates within the formal structure of the theory.

On the other hand, the postulate of the constancy of the speed of light decrees (among other things) a certain systemic isotropy, namely, that all light signals propagate at the same speed in all directions, for all states of the emitting body. Obviously, the nomological isotropy of the laws that is assumed in the derivation of the Lorentz transformations is insufficient to guarantee this particular property of light. Even when allied with the principle of relativity and space-time homogeneity, we only obtain that *some* velocity $k^{-1/2}$ must exhibit such invariance, but this limit velocity need not be related to light (though of course it turns out to be so in our world). Conversely, the assumption that the speed of light possesses this systemic isotropy is not in itself enough to guarantee that the transformations themselves will exhibit nomological isotropy. Indeed, in (Drory, 2014), I obtained anisotropic transformations which nevertheless embody the isotropic propagation of light signals. These transformations are not nomologically isotropic, but light propagation remains systemically isotropic under them.

Here again, it is important to note that the systemic isotropy of light is postulated, and that the question of its empirical adequacy is not relevant to the present discussion. Thus, general relativity limits this systemic symmetry to flat regions of space-time, but this only means that the second postulate, and hence special relativity itself, has limited applicability. The question under discussion here, however, assumes that the postulates are correct and seeks to determine the logical structure of their relations.

Though it might appear trivial at this point, the distinction between these two meanings of “isotropy” will be helpful in the remainder of the present analysis. As we approach the problem of determining empirically the value of k , we shall have to consider problems relating to isotropy. In every such case, we shall mean the isotropy of specific effects or specific apparatuses meant to measure k . The isotropy required in every such case is always systemic, and it will be important to remember that such a symmetry is not guaranteed by the nomological symmetry postulated in the derivation of the generalized Lorentz transformations. Instead, if a suggested empirical procedure for measuring k depends on the property of isotropy (as some will prove to in the next sections), it will have to be analyzed on a per case basis. In no case is the isotropy referred to in such cases, which is clearly systemic, derivable from the nomological isotropy postulated in the theory.

6. Synchronization redux

Let us now return to the synchronization procedure. We wish to ensure that two clocks are moved at identical speeds but in opposite directions by building identical moving mechanisms and then inverting one of them. But what type of isotropy are we invoking here when we assume that the motor built according to rotated blueprints will perform the function we seek?

Apparently, we are using nomological isotropy since we are merely rotating the axes of the coordinate system. This seems to solve the problem until we reflect that when we speak of the moving mechanism, we must consider everything that can influence the motion of the clock. That must include every object that *might* exert a force on the clock. Rotating the motor and rails is not enough. If we want to include every *possible* influence, we must rotate the whole universe. In principle, the gravitational influence of a single star might affect the motions of the clocks by speeding up the velocity of the clock that moves towards the star, while braking the one that moves away from the star. One might like to say that such influences are negligible, but unfortunately, we cannot make such a claim without knowing beforehand something about the value of k . If k is small enough, the measurement might be influenced by extremely minute differences in the configuration of the external world. Recall that we are not privy to any details about the value of k save the assumption that it is finite. Nothing about its actual value can be assumed. Besides, in a gedanken setting precision is absolute, so even small influences must be taken into account.

One may object that the Einstein synchronizing procedure is vulnerable to the same problem because gravitational influences affect the speed of light. The anisotropy of the world itself should impact the use of light signals to synchronize clocks, therefore. But our present concern is with special relativity, where light is *postulated* to move at a universally constant speed. That the second postulate has limited validity in our world does not influence the logical structure of the theory. It is important to realize that the second postulate is exactly that, a postulate. We do not get to argue with its validity, in the sense that special relativity will only hold whenever the second postulate holds. Whether the

second postulate is true in our world is indeed an empirical matter, and must be determined as such. This determination does not replace the postulate itself. The constancy of the speed of light is an assumption that belongs to the logical structure of the theory; its empirical verification belongs to the realm of the theory's confirmation.

The situation is completely different in the case of the mechanically moving clocks in Mermin's procedure. No additional postulate *logically* "protects" the mechanisms from outside influences. The problem here is not that we assume something about the workings of the transportation mechanisms and then face the question of its empirical adequacy. Rather, the problem is that no additional assumptions are made at all. Nothing allows us to logically expect that rotated mechanisms will work identically if we do not rotate everything else around them. The reason is that it is unclear what additional assumptions are necessary to ensure that the motions of the two clocks are identical. It is not enough, for example, to postulate that gravitational influences are negligible. That would make the situation similar to special relativity, where we postulate a condition that we know to be violated in the presence of strong gravitational fields. But here gravitational influences are just one of infinitely many possible influences. We need a complete theory of mechanics in order to know what is relevant and what is not in constructing the rotated machines. Thus, the kinematics of the theory, which is where we find the parameter k , depends on its dynamics, in a manner quite unlike what happens in special relativity, where a single assumption covers every contingency. Furthermore, we expect that the dynamics will be founded on the kinematics rather than the other way around. The only non-dynamic assumption that will make Mermin's synchronization procedure work is if the structure of the entire universe is isotropic down to local details. While logically possible, such a postulate is problematic for being obviously violated in our world, as even a cursory observation will reveal, not to mention that were we even considering the adoption of such a postulate, we would lose any logical advantage we supposedly gain from working with the principle of relativity alone.

Clock transportation methods must therefore be considered inadequate. But if there must be no transportation involved, we must use some signal to transfer information between the various clocks and the observer, or between the clocks themselves. This implies that we must make some assumptions regarding such signals.

Suppose for starters that we use material agents as signals. The clocks could be equipped with some sort of gun that fires pellets at a certain velocity, v_0 . To synchronize two clocks, the observer positions himself midway between the two clocks. The clocks are set so that each fires a pellet at a predetermined time, say when each clock marks the time zero. If the pellets reach the observer simultaneously, the clocks are considered synchronized. If not, one (or both) clocks are adjusted and the procedure repeated until they are synchronized. To synchronize a third clock, the observer moves and positions himself midway between clock 2 and 3. In the repeated procedure only clock 3 will be adjusted until it is observed to be synchronized with clock 2. In the same manner any

number of clocks can be synchronized with each other.

But this method has the same weakness as the transportation procedure. We must be certain that the pellets are fired at equal speeds. Since the clocks must fire the pellets in opposite directions, however, we cannot determine directly that the pellets move identically. If we build identical guns and rotate one of them, we lose in the process the assurance that nothing has influenced the workings of the firing mechanism; if we build the guns as rotated images of each other, we must make sure that every possible external influence has also been rotated. Instead of pellets, we could use some mechanical devices running from the clocks to the observers, but again the same criticism applies. There is no way to be certain that the mechanical messengers run in opposite directions at identical velocities without additional postulates.

Mechanical signaling being unreliable, we might try non-material signals as a means of synchronizing a clock network. This already requires a certain amount of knowledge, for it is not clear how we determine which signals are material and which are not. It was not immediately evident to the pioneers of radioactivity that alpha and beta rays were material particles but gamma rays were not. To make such a distinction already requires entering into the details of the signal properties.

Even if we do not care whether the signals are material or not, we still require significant knowledge about them in order to use them as synchronizers. Suppose for example that we decided to use sound waves to synchronize our clocks. To do this, we must be aware that sound waves travel in air and that their velocity is simply defined only with respect to that medium. In turn, this requires knowing the state of motion of air with respect to us. For example, we should make sure that we perform the synchronization inside a sealed off container, to make sure that the ambient air travels with us (with respect to whatever reference frame we wish). This is a necessary condition to ensure that the signals travel at equal speeds from the two clocks. In an open container, the air might appear to flow with respect to us at a velocity V . That would introduce a spatial asymmetry in the system, and sounds emitted in the direction of flow of the air would travel (with respect to us) at a velocity different than those emitted in the opposite direction. Their velocities would no longer be equal and any synchronization procedure would then have to take this into account. This can be done, of course, but the velocity of sound with respect to air will then appear explicitly in the equations. Without knowing this quantity, one could not therefore correct for the anisotropy of the signal's propagation. All these details would have to be added to the theory as separate assumptions. Relegating those to the realm of "empirical evidence" is no different than stating that the constancy of the speed of light is an empirical fact. If we intend to claim that a second postulate is unnecessary, we cannot introduce in its stead highly detailed information about other signal propagations.

Even using a closed container will not suffice, however. We need to make sure that the air is in identical conditions everywhere. Knowing that the velocity of sound depends on the density of the material might be too much specific information to assume, but we clearly cannot assume that the velocity of the

signal is independent of ambient conditions. We shall face very quickly the same problem we encountered with pellets. Unless we can ensure that everything that might influence the state of the medium of propagation is uniformly distributed or at least invariant under rotation, we have no way of ensuring that our synchronization procedure is sound.

Using light instead of sound makes no difference. We need to know with respect to what “medium” or “system” the speed of light is identical in both directions and what our velocity is with respect to that “medium”. Of course, Einstein’s second postulate is just a way of providing this information, by denying that there is any particular and unique medium with respect to which the velocity is c .

Thus, we see that creating a network of synchronized clocks is a highly non-trivial procedure, and one that requires additional assumptions beyond just the principle of relativity. Either we assume that $k = 0$ (a result that cannot be established in actual experiments), or we assume something about signaling methods, whether mechanical or wave-like. In either case, we shall need other postulates, and none are as simple and elegant as the postulate of constancy of the speed of light.

The question of determining the value of k , thereby completing the transformation equations, now depends on whether we can find k without the need for a network of synchronized clocks. The methods presented in the previous sections all required synchronized clocks. Let us turn now to options that manage to bypass this need.

7. Finding k from the Velocity Addition Formula

N. David Mermin suggested a way to obtain the value of k without using clocks at all (Mermin, 1984). His highly ingenious proposal is based on the use of the velocity addition formula, Eq.(2). Mermin’s original exposition was not meant just to find k and it contains many extraneous details, therefore. Because of this, I shall recap it here and adapt it to the problem at hand.

Suppose that an observer A wants to measure the velocity of an object B. Such measurements involve a choice of units. Instead of using units of length and time, one can directly use units of velocity by comparing the velocity of B to that of a unit runner (a “hare” in Mermin’s terminology). We assume that A has an object that can move at a velocity u_A on a straight track for a certain length L , then stop and run back at the same speed in the opposite direction, i.e., at a velocity $-u_A$ with respect to the observer A. Let us also assume that B moves more slowly than the unit runner, so that the unit runner reaches the end of its track and doubles back before B has reached the end of the same track. We also assume that both B and the unit runner are at identical positions at some moment, that this moment is chosen to be $t = 0$ and that the position is chosen to be the beginning of the unit runner track.

Let t_1 be the time at which the unit runner reaches the end of its track, so that

$$u_A \cdot t_1 = L \quad (7)$$

The runner turns around when the body B is still moving in its original direction (towards the end of the runner's track). Let t_2 be the time that elapses between the moment the runner turns around and the moment it meets the body B again. Clearly, we have that:

$$v_{BA}(t_1 + t_2) + u_A t_2 = L, \quad (8)$$

where v_{BA} is the velocity of the object B in the frame of reference A. The meeting between B and the unit runner takes place a distance L_0 from the track's end, where

$$L_0 = u_A t_2 \quad (9)$$

Mermin then notes that the ratio $\frac{L_0}{L}$ is a frame invariant quantity. For example, one can imagine the distance L being broken into N separate length units numbered from 1 to N . To every distance L_0 corresponds a number n representing the particular unit where the meeting happens. This must be agreed on all observers, because we could have the segment n light up when the meeting happens while all the other segments remain turned off. As N grows to infinity, the ratio $\frac{n}{N}$ approaches the value of $\frac{L_0}{L}$, which must also be frame invariant, therefore. From Eq.(7) and Eq.(9), we can isolate t_1 and t_2 , then substitute for these quantities in Eq.(8). The result is:

$$\frac{v_{BA}}{u_A} = \frac{L - L_0}{L + L_0} = \frac{1 - \left(\frac{L_0}{L}\right)}{1 + \left(\frac{L_0}{L}\right)} \quad (10)$$

Hence, we can measure the velocity of B in terms of the unit runner velocity, u_A , without the need for clocks or rulers. We only need the frame-invariant and unit-independent ratio $\frac{L_0}{L}$.

To measure the value of k , we now proceed in the following manner. Let A and B be two frames moving in parallel in the same direction. Each has a unit runner moving with respect to its own frame at a velocity u_A and u_B respectively. The runners need not be identical, since only velocity ratios will enter the final expression. Thus, A and B can choose any velocities u_A and u_B they please. Without loss of generality, let us assume that $u_A > u_B$, however. Now let an object C move parallel to the direction of motion of B with respect to A. Using the unit runner method, A now measures the two ratios

$$\begin{aligned} r_B &= \frac{v_{BA}}{u_A} \\ r_C &= \frac{v_{CA}}{u_A} \end{aligned} \quad (11)$$

i.e., A measures the motion of the object C and of the frame B in units of its runner's velocity. Similarly, B measures the velocities of A and C with his own unit runner and finds the ratios

$$\begin{aligned} s_A &= \frac{v_{AB}}{u_B} \\ s_C &= \frac{v_{CB}}{u_B} \end{aligned} \quad (12)$$

We can now use the velocity addition rule, Eq.(2), to obtain:

$$v_{CA} = \frac{v_{CB} + v_{BA}}{1 + kv_{CB}v_{BA}} = v_{BA} \left[\frac{1 + \left(\frac{v_{CB}}{v_{BA}}\right)}{1 + k \left(\frac{v_{CB}}{v_{BA}}\right) v_{BA}^2} \right] \quad (13)$$

Finally, we note that $v_{BA} = -v_{AB}$. This lets us rewrite

$$\frac{v_{CB}}{v_{BA}} = -\frac{v_{CB}}{v_{AB}} = -\frac{s_C}{s_A} \quad (14)$$

and Eq.(13) then becomes

$$\frac{r_C}{r_B} = \frac{1 - \left(\frac{s_C}{s_A}\right)}{1 - ku_A^2 \left(\frac{s_C}{s_A}\right) r_B^2} \quad (15)$$

From this we obtain finally,

$$ku_A^2 = \frac{1}{r_B r_C} \left[1 - \frac{s_A}{s_C} \left(1 - \frac{r_C}{r_B} \right) \right] \quad (16)$$

This expression yields the invariant velocity $k^{-1/2}$ in terms of the unit runner velocity of the A frame, u_A . This value is expressed entirely through dimensionless frame-invariant ratios, for which we need neither clocks nor rulers at all.

But just because we do not need directly any synchronized clocks does not mean that we are out of the woods. In section 4 we considered a synchronization method involving two clocks signaling to an observer placed midway between them. The problem we came up against was how to make sure that the signals move at identical speeds to the left and to the right. While Mermin's method does indeed bypass the use of clocks altogether, it still requires solving the exact same problem. For the analysis of the present section to work, we must be sure that after the unit runner reaches the end of its track and turns around, it moves with the same speed that it had before. Indeed, Mermin notes correctly that the unit runner (a "hare" in his terminology) must be attached to "*a mechanism of propulsion that couples each hare to its track in the same way for the right-left and left-right parts of its race*" (Mermin, 1984, p. 124).

But if we could create such a mechanism, we could use it to synchronize clocks. Thus, the supposed superiority of Mermin's method for finding k , i.e., that it uses no clocks, is in fact illusory, for if we could implement his method, we could also create a network of synchronized clocks and use any of the previous methods in order to measure k .

The situation here is in fact worse, because while clock synchronization requires moving two objects (such as signaling pellets) at identical speed in opposite directions, Mermin's method of determining k requires that the left-to-right and right-to-left motions not only be identical, but also uniform. For signaling, it does not matter whether the pellets' velocity is constant, as long as it is identical in both directions. Here, on the other hand, Eq.(8) only holds for constant speeds. If the speeds are varying, we need to know their precise dependence on time in order to find the relation between $\frac{L_0}{L}$ and k . It is unclear then that we could extract k from merely the ratio $\frac{L_0}{L}$, and we would likely need more parameters.

The problem is how we can know that the unit runners are moving uniformly. In Mermin's analysis, the possibility of creating a moving mechanism that generates a uniform motion is simply assumed. But it is not evident how to do this (in the gedanken sense of the term). If we try to devise such a mechanism theoretically, we need a complete theory of mechanics so that we know what forces are at work and how they influence the runner's motion. Since we are in a gedanken setting, we are not allowed to dismiss any influence as negligible. We are imagining absolute precision, and even a distant star's gravitational influence has to be taken into account. The problem is of course that we cannot know all the influences, nor do we know exactly what the dynamical laws that determine these influences are. All this is beyond the scope of special relativity, and to require such complete knowledge in order to make sense of the theory is unreasonable.

We do not need to devise the runner's mechanism perfectly theoretically, though. We could in principle adjust the mechanism by trial and error (again, possible because of the gedanken setting) until it actually causes the runner to move uniformly. In order to achieve this, however, we need a way to determine whether the runner's motion is indeed uniform. Without it, we cannot know how to adjust the mechanism or how successful we have been.

The most natural way to verify the constancy of the runner's velocity would be to check that for any set of equal time intervals Δt , the runner advances equal distances, Δx . But this requires a network of synchronized clocks that allows us to determine the position of the runner at equal time intervals Δt . Such a network is impossible to create, however, without the postulate of the constancy of the speed of light, or something that can replace it. Thus, we are once again in the position that a second postulate is required to ensure that we can set up an experimental determination of k . Of course, one may think that there may be methods of determining the constancy of a velocity without the need for synchronized clocks. I shall examine such a proposal in section 9 and

show there why it fails as well.

There remains one possibility, though, since we do not need to know what the velocity of the runner is, but merely require that it be uniform. Because it involves the concept of inertial frames, the principle of relativity itself is based on one mechanical law that we must accept as part of the first postulate, namely, the principle of inertia. By this principle, we know that a body moves uniformly if it is isolated from other bodies. I shall not enter here into the debate regarding the logical structure of the principle of inertia. Let us just assume, for the sake of the argument, that if we have an isolated body, one far enough from any other body (whatever that means), this guarantees that it is under no velocity-changing influence, and let us further assume that such bodies can be found. If we could use such a body as the runner of a reference frame, we would at least guarantee that its motion is uniform (but note that this would still not solve the problem of having it reverse its motion and keep the same speed).

Unfortunately, this would merely shift the focus of our problem without solving it. For now, although we have a proper runner, it must remain isolated from all other bodies. Yet, the procedure we are discussing here requires this runner to cross paths with two other moving bodies. Clearly we cannot allow the runner to physically cross paths with these other bodies; indeed the runner must remain extremely far from them. Even worse, it must remain far from the measuring mechanisms that we use to determine where it meets the other bodies. All this means that the runner must signal us where it is at any time. But then we are back to the old problems again. We must know the properties of the signal in order to know where the runner meets the other bodies. Since the signal must take a finite time to reach us, we must know this time in order to calculate when it was emitted and determine the runner's position at that time. Even the moment at which the runner meets the bodies whose velocities we are measuring must be determined by signaling, since the runner cannot be in physical proximity to these objects. If we rely on the runner's isolation to make sure it is moving uniformly, then every phase of the measurement must involve some form of signaling. The properties of the signal are once again central to the experiment, and we are back to some form of the second postulate. Without knowing, for example, that the velocity of the signal is independent of the emitter's speed as it is in standard special relativity, or some other adequate information, we cannot calculate the actual positions of the various objects from their observed positions. Any other assumption needed to perform these calculations would function as some sort of second postulate in the theory. Once again, we see the necessity for such a postulate.

Still, there is one last possibility of finding k that seemingly does not require a set of synchronized clocks. I shall now turn to it.

8. Finding k from the Doppler Shift

Since $k^{-1/2}$ has the units of velocity, any determination of its value must be relative, i.e., we compare $k^{-1/2}$ to some other velocity chosen to be a standard unit. In the previous section, we measured ku_A^2 , where u_A is the velocity of

some standard object, specifically a mechanical runner. Another option is to compare $k^{-1/2}$ with the speed of some signal that could serve as a standard speed. One possibility of determining this without the need for synchronized clocks involves the comparison of two periods, using the Doppler effect.

Suppose that we wish to measure the velocity v of an object moving in the positive x direction towards an observer S. The body emits a periodic signal that also travels in the positive x -direction. Let the frame S' be the rest frame of the body, so that it moves with respect to the frame S with a velocity v . Let us further assume that the rest frequency of the signal is f_0 , so that with respect to the moving frame S', the period is $T_0 = \frac{1}{f_0}$.

The signal generator is moving with respect to the S frame, however, so that in that frame it emits a signal with a period

$$T = \frac{T_0}{\sqrt{1 - kv^2}} \quad (17)$$

in accordance with the generalized time dilation rule.

Let the velocity of the signal in the S frame be c (we assume nothing regarding its velocity in the S' frame). At a certain moment, the moving body emits a wave crest, for example. By the time the emitter sends the next crest, the first one has moved a distance $cT = \lambda$ towards the observer. But the emitter has also advanced, by a distance vT . Thus, the distance between one crest and the next is only

$$\lambda_{obs} = (c - v) T \quad (18)$$

The period observed by S is the time between these two crests, which is

$$T_{obs} = \frac{\lambda_{obs}}{c} = \left[1 - \frac{v}{c}\right] \frac{T_0}{\sqrt{1 - kv^2}} \quad (19)$$

The observed frequency is therefore

$$f_{obs} = \frac{1}{T_{obs}} = \frac{\left[1 - \frac{v}{c}\right]}{\sqrt{1 - (kc^2)\frac{v^2}{c^2}}} f_0 \quad (20)$$

In this equation, the generalized Doppler formula, all the velocities, including the invariant velocity $k^{-1/2}$, are expressed relative to the signal velocity c that can be chosen as a unit velocity.

To measure the ratio $\frac{f_{obs}}{f_0}$ (or equivalently, $\frac{T_{obs}}{T_0}$), the observer S needs only one clock by which he measures the frequencies emitted, once by the body in motion and once by an identical signal emitter at rest. There is no need to synchronize this clock with another. The measured ratio $\frac{f_{obs}}{f_0}$ contains the desired parameter kc^2 .

Unfortunately, it also contains the emitter velocity ratio, $\frac{v}{c}$. Because c has already been selected as the unit velocity, $\frac{v}{c}$ can no longer be chosen as unity and must be determined as well. Thus, Eq.(20) contains two unknowns and only one measured quantity, the frequency ratio, to determine them. This means we cannot extract the desired parameter kc^2 unless we can find another equation involving the same unknowns (and only them).

One possibility is to reverse the direction of motion of the object, so that $v \rightarrow -v$. This would mean measuring the frequency emitted by an object moving away from the observer. Eq.(20) would then become

$$f_{obs,2} = \frac{\left[1 + \frac{v}{c}\right]}{\sqrt{1 - (kc^2)\frac{v^2}{c^2}}} f_0 \quad (21)$$

Together with Eq.(20) we could now extract the value of kc^2 . But of course, we are facing the same problem that we met several times already: How can we make sure that the object will move with a velocity $-v$? This is the same difficulty we encountered in synchronizing clocks and in Mermin's unit runner procedure. As in the latter case, we may not directly need synchronized clocks, but the preconditions of this experiment would solve the problem of synchronization as well. Thus, the two problems are equivalent.

A seemingly better choice is to keep the body moving in the same direction, but repositioning the observer behind it. The effect is the same, namely, that the signal is now emitted from a body moving away from the observer, but no mechanical inversion took place, so the body keeps moving as before. Unfortunately, the signal does not. We would like to claim that this situation is related to the previous one (the body moving towards the observer) by performing the simple replacement $c \rightarrow -c$. That would yield once again Eq.(21). But we cannot, because nothing guarantees that the signal's speed is the same when the emitter moves away from the observer as it was while moving towards him. Indeed, if we use sound waves and the observer is not in the rest frame of the propagation medium (e.g., air), we know that these two speeds will be different. What we must do is then to assume one speed for motion towards the observer, say c_+ , and another for motion away from the observer, say c_- . In this case, the two equations we obtain for the observed frequency ratios are:

$$\begin{aligned}
\frac{f_+}{f_0} &= \frac{\left[1 - \frac{v}{c_+}\right]}{\sqrt{1 - (kc_+^2) \frac{v^2}{c_+^2}}} \\
\frac{f_-}{f_0} &= \frac{\left[1 + \frac{v}{c_-}\right]}{\sqrt{1 - (kc_-^2) \frac{v^2}{c_-^2}}}
\end{aligned} \tag{22}$$

This renders the method useless, of course. While the two ratios on the left hand sides of the equations can be measured, we have four unknowns, $\frac{v}{c_+}$, $\frac{v}{c_-}$, kc_+^2 and kc_-^2 , which still leaves us without possibility to determine k . Of course, Einstein's second postulate is all about solving this problem, by proclaiming as an assumption that $c_+ = c_-$, which reduces the number of unknowns to the required two. Yet this is precisely the assumption that we seemingly want to avoid. Once again, we are at a dead end.

9. Inertiality of Frames

Above and beyond all the concerns dealt with in the last sections, there is one that goes to the heart of special relativity theory. That theory only deals with inertial frames. How can we make sure then, that the frames from which and to which we are transforming are actually inertial? This problem is closely connected to the concern raised at the end of section 7, which described Mermin's approach. There I noted already the difficulties associated with making sure that an object moves uniformly. Clearly this is the same problem because in order to make sure that a frame is inertial we must verify that an isolated body moves at a constant velocity. Even given that we can bypass this for one specific frame, by some sort of fiat, or separate assumption, still the criterion for another frame to be also inertial is that it moves uniformly with respect to the first. How can we make sure that this the case?

As discussed in section 7, this problem already plagues Mermin's method of determining k . We cannot devise a mechanism that will move something uniformly on a purely theoretical basis because that would require knowing all the laws of dynamics and the disposition and properties of every object in the universe. Instead, we must find a method that will let us determine when an object moves at constant velocity. We can then adjust whatever moving mechanism we devise until it generates the proper motion.

The problem is how to *check* that a body moves uniformly, therefore. The natural method, namely measuring that equal distances are traveled in equal times, requires a network of synchronized clocks, which itself requires something

akin to the postulate of constancy of the speed of light. The previous section offers an alternative, however.

Let us attach to the body whose motion we wish to check a wave emitter. While the body is moving, e.g., towards us, we check as before the observed frequency of the signal. Eq.(20) shows that the ratio of that frequency to the rest frequency of the emitter depends on the velocity v of the body. For the present purpose, we do not need to know what that velocity is. The question is whether we can use this as a criterion for the velocity being constant. At first sight it appears that we can, because if v varies in time, we expect that the observed frequency should also vary. Thus, if the observed frequency remains constant, we would like to deduce that the body's velocity must also remain constant.

Unfortunately, a look at the Doppler shift equation reveals that once again the second postulate is necessary to validate such a conclusion. Let us assume that empirically, the ratio of the observed frequency to the rest frequency remains constant in time. Let us denote this constant by $\alpha = \frac{f_{obs}}{f_0}$. Then, from Eq.(20) we have that

$$\alpha\sqrt{1 - kv^2} = 1 - \frac{v}{c} \quad (23)$$

Now we would like to deduce from this that if α is constant in time, so is v . This is only true if c is also constant, however. This is because Eq.(23) is a quadratic equation for v in terms of k , α and c . k is constant from the derivation of the generalized Lorentz equations and we assume that α is constant in the specific process we are observing. But if c is not constant in time, we cannot deduce that the velocity v is constant. Indeed, from Eq.(23) we can obtain the general relation

$$c = \frac{v}{1 - \alpha\sqrt{1 - kv^2}} \quad (24)$$

Thus, the frequency ratio α can remain constant even if the velocity of the observed object varies, provided the velocity of the emitted signal, c , also varies with the velocity of the object in accordance with Eq.(24). Without the second postulate we have no reason to think that c cannot vary with the velocity of the emitter, and this possibility cannot be discounted.

One should note that Eq.(24) does not represent an explicit functional law that stipulates the relation between c and v , because we do not know how the frequency ratio α depends generally on the velocity of the emitter. It is true that we assume that α is constant, but that does not mean that it is independent of v in general. On the contrary, we would expect it to depend on the velocity of the emitter, if for no other reason that as $v \rightarrow 0$, we expect that $\alpha \rightarrow 1$, but it is certainly different than unity in other cases. We cannot tell how α depends exactly on v , however. The Doppler shift formula only relates α to v and c . If c varies with the velocity, we cannot tell what is the precise relation of α to v without specifying how exactly c depends on the velocity. In other words,

Eq.(24) relates two unknown functions of the velocity, α and c . In principle, almost any function of v that approaches unity as v approaches zero is a possible description of α , and yields a valid relation between c and v . I say almost because such a function must represent a law of nature, and must therefore be covariant under the generalized Lorentz transformation. Such functions abound, however, and the principle of relativity is insufficient on its own to select a unique relation between α and v , or equivalently between c and v .

One may object that the relation implied by Eq.(24) is unlikely. Indeed, it seemingly is, because as $v \rightarrow 0$, we know that $\alpha \rightarrow 1$, so that $c \rightarrow 0$. This means that an object at rest cannot emit a signal, a stipulation at odds with even the most cursory observation. But Eq.(24) need not be a general law in order to invalidate the Doppler test of the constancy of v . It suffices that for *some* range of velocities, the signal speed should vary as Eq.(24) requires. It may well be that as v approaches zero, the functional relation between c and v changes completely. Yet if the relation Eq.(24) holds for some velocity range, it is enough to prevent us from deducing that v is uniform when we observe that α is constant in time, because we have no way of knowing whether the specific process we are observing does not in fact take place precisely within that range. Thus, we have no grounds whatsoever to eliminate the possibility that Eq.(24) holds for any specific process we observe, even if it does not hold for all possible processes. This means that we cannot use the Doppler shift to make sure that our body moves uniformly.

As mentioned in section 7, this leaves us with only one possibility, and that is to rely on the principle of inertia, which is built into the principle of relativity. As I argued above, however, this requires that the two frames we are comparing must be very distant from one another, otherwise the conditions of the law of inertia will not hold. If that is the case, comparisons of space and time measurements between the frames necessitate some signaling between them. Such signaling is a necessary precondition to give the transformations some physical meaning, therefore. The equations refer to a single event as determined from two frames. Yet if the observers - meaning here the actual physical entities that perform the measurements - are widely separated, at least one of them (or both) must also be far from the observed event. An observation of the event can then be done only by having some information transmitted from the event to the measuring devices, or from the measuring devices to the physical observer. Whatever the phase of the process in which we must transmit information between two widely separated locations, such transmission must take place if we are to acquire the space and time coordinates to which the transformations refer. Now any such signal takes time to travel and we must know something about its propagation, therefore. Without some additional assumption akin to the second postulate, we cannot know what the actual space and time coordinates of the event are, i.e., those very quantities that lie at the basis of the theory.

The need for the second postulate lies at the core of the theory, therefore. The very objects to which it applies, inertial frames, space and time coordinates, cannot be established or measured without some recourse to a method

of signaling with known laws of propagation. Without the second postulate, we cannot make sense of the very things with which the theory is concerned.

10. The essence of the second postulate

The previous section brings us close, finally, to understanding the function (or at least one essential function) of the second postulate. Recall Lévy-Leblond's opinion, quoted in section 1, that the properties of light are over-emphasized in SR, an opinion later echoed by Mermin and others. Yet we have seen that something goes awry when we try to imbue the transformations with complete physical meaning. It seems that we hit a snag every time we try to perform experiments and measure k . In fact, the situation is worse for not only the measurement of k seems to require additional postulates, but almost all measurements are in trouble, because they must rely on a network of synchronized clocks and identical rulers. Setting up such a network runs into trouble without additional postulates.

Why should that be? Is there any fundamental reason why we should need such contingent information as describes the propagation of some specific signal? Note that I do *not* claim that SR need be based on light, as opposed to some other signal, and in this sense (but only in this sense) I agree that SR is not necessarily related to electromagnetism. Other postulates could work equally well to establish the prerequisites of space-time measurements. But neither can special relativity be said to follow from space-time symmetries and the principle of relativity alone.

The general direction seems to me to be this. At its deepest level, special relativity can be thought of as a theory that unifies space and time into space-time, with a four-dimensional metric. Thus, special relativity revises our understanding of the geometry of space and time, which is essentially the relation between different points in space-time. But an observer is always a local entity, in fact a point-like entity. In the end, all observations and measurements must reach this specific point for analysis. There is therefore a fundamental gap between the referent of the theory - which relates different space-time points - and the observer, which is always located at a single point. In order for the space-time measurements to be properly compared and related, such as in the generalized Lorentz transformations, there must be some way to transfer the information from various space-time points to the single location of the observer. It is precisely this transfer of information that is the common ground of all the problems we have encountered in the previous sections. Transporting clocks, shooting pellets, signaling, all share one fundamental aspect - they are methods of transferring information from one location to another. The necessity for such a method is intimately linked to the fact that special relativity is a geometric theory, one that is concerned with the relation between separate points of space-time.

This is why the mathematical form of the Lorentz transformations is useless *per se*, unless we have a means to determine the value of k . And determining this requires some transport of information. That it must be so is therefore

fundamental to the meaning of special relativity as a theory of space-time structure. And it is the second postulate that allows us to analyze such information transmission. Without it, the theory is literally blind. In any physical sense, therefore, the special theory of relativity is truly based on space-time symmetries and *two* postulates, rather than just on the principle of relativity.

We saw repeatedly that a fundamental precondition of experiment is having a signal (material or otherwise) that moves identically in opposite directions. The second postulate ascribes such a property to light, of course, but it apparently prescribes more - namely that the value of its speed should be invariant in all inertial frames. Is any additional physical information conveyed by this logically stronger requirement? The answer is negative. In addition to the postulates A-C, i.e., space-time homogeneity, spatial isotropy and the principle of relativity, one need add only the following postulate in order to derive standard SR:

Postulate of Light Isotropy: A light signal propagates with the same speed in all directions, no matter the state of motion of its source.

Although logically weaker than Einstein's second postulate, this assumption does in fact imply that the speed of light is frame-invariant. The proof is very simple. Consider a light source at rest in the frame S, which emits two light signals in opposite directions, say the positive and negative x direction. Let the velocities of the two signals be denoted by c and $-c$ respectively.

In a frame S' moving with velocity v with respect to S, the signals will appear to move with velocities c'_+ and c'_- respectively. The postulate of light isotropy now implies that $c'_- = -c'_+$. These velocities can be calculated from the rule of addition of velocities, Eq.(2). From the requirement $c'_- = -c'_+$, we then have:

$$c'_- = \frac{v - c}{1 - kcv} = -c'_+ = -\frac{v + c}{1 + kcv}; \quad (25)$$

From this equation we immediately obtain:

$$k = \frac{1}{c^2} \quad (26)$$

which also immediately implies that $c'_+ = c$. Thus any signal that verifies the postulate of isotropic propagation must propagate at the invariant speed $k^{-1/2}$. The present analysis directly contradicts, therefore, Mermin's assertion, already quoted in section 1, that:

It is not, however, necessary for there to be phenomena propagating at the invariant speed to reveal the value of k . (Mermin, 1984, p. 123)

On the contrary, measuring k (as well as, e.g., creating a network of synchronized clocks) absolutely requires knowing which signal propagates at speed $k^{-1/2}$, and *a fortiori*, that such a signal exists.

This signal plays a fundamental role in relating different points in space-time. Such a transfer of information is a necessary precondition of any experiment and is necessary, in general, to make physical sense of the theory in a way that

Newtonian physics does not require. Hence, some sort of specific information about signals must be part of the fundamentals of the theory, and the isotropy postulate is the core information required.

11. Summary

The content of innovative physical theories lies not just in new equations. They often involve redefining previously established concepts, revising fundamental assumptions and at their best, reforging our world-view. Once such a theory has been accepted by the scientific community and has become established, it can be easy to view these other changes as independent of the theory. At this point, there is often a tendency to axiomatize the theory and to seek its simplest possible mathematical formulation. There is much to be gained from such attempts, but in the process, we are facing a danger of over-formalization.

Special relativity is almost unique in being a theory that emerges from a revision of the preconditions of experiments. It is intimately linked to the question of how we measure time and space coordinates and what the necessary preconditions for such measurements are. Such analysis is so much a part of our world-view nowadays that we may think it can be disconnected from other aspects of the theory. I venture no opinion whether this is the case or not, but I do argue that the importance of the postulate of the constancy of the speed of light does not lie only (or even mostly) in the formal completion of the equations. The postulate is much more than merely a claim regarding the value of some free parameter in the theory. Instead, it plays an important part in the analysis of the preconditions of experiments that form the core of special relativity.

In this paper I have analyzed what it means to empirically determine the value of the parameter k . Direct use of the transformations to extract k from length contraction or time dilation fail because they require a set of synchronized clocks as a prerequisite to the measurement. Procedures for synchronizing clocks must rely on either signaling or mechanical transportation (either of the clocks themselves or of some material carriers of information). Both these methods fail if further assumptions are not added. These assumptions may be dynamical (in the case of mechanical transportation) or kinematical (assumptions about the velocity of non-material signals). In either case, they amount to a second postulate of some kind. Among possible postulates, it appears to me that the Einstein postulate is particularly elegant and simple, though this may be a personal preference.

Methods that do not require synchronized clocks still fail without additional postulates. These could in turn be used to create a synchronized network of clocks. Mermin's method of unit runners, for example, requires ensuring that some device moves identically to the left and to the right, a construct that would allow clock synchronization. I have argued that we have no way to ensure such symmetry without additional assumptions, however. Precisely the same problems plague attempts to use the Doppler effect to determine the value of k .

Finally, these last methods require not only that left and right motions be identical, but also that they should be uniform. This problem reaches to the very essence of special relativity as a theory concerned with inertial frames only. The identification of a frame as inertial requires the ability to check whether a motion is uniform. All methods for achieving this either require a network of synchronized clocks or constancy of a signal's velocity (as in the case of the Doppler shift method). The only method that ensures automatically uniform motion is based on the principle of inertia but that requires physical separation between the frames themselves and/or between them and the observed event. Such a separation requires that all information must be obtained via signaling. To reconstruct the correct physical quantities then requires some detailed information about the propagation of the signals used, which must be postulated separately.

The required information is logically weaker than Einstein's assumption of constancy of the speed of light. This is the postulate of isotropy of light speed, which states that the velocity of light is always identical in all directions, no matter the state of the emitter. By adding this postulate to the principle of relativity and the space-time symmetries, one recovers the Lorentz transformations together with their physical underpinnings.

This shows that the essence of the second postulate lies in providing us with a phenomenon that propagates with identical speeds in opposite directions. This function, however trivial, contingent or specific it might seem initially, turns out to be crucial to the physical content of the theory.

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